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# Reliability analysis using an enhanced response surface moment method

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## Abstract

Moment methods, which are powerful and simple techniques for analyzing the reliability of a system, evaluate the statistical moments of a system response function and use information from the probability distribution in the analysis. The full factorial moment method (FFMM) performs reliability analysis by using a $3^n$ full factorial design of experiments (DOE) and the Pearson system for random variables. To overcome the inefficiency of FFMM, the response surface moment method (RSMM) has been proposed, which is based on a response surface model (RSM) that is updated by adding cross product terms into the simple quadratic model. In this paper, we propose the enhanced RSMM (RSMM+) that modifies the procedure of selecting a cross product term in the RSMM and adds a process of judging whether the response surface model can be established before performing an additional experiment. We apply the proposed method to several examples and show that it gives better results in efficiency.

*Keywords*: Reliability analysis; Moment method; DOE; Pearson system; RSMM

## 1. Introduction

Engineering design consists of three steps. First, the design requirements for shapes and performances are determined, considering the needs of consumers. Second, the design variables that tune the system responses are set up. Last, the values of design variables that satisfy the design requirements are suggested. After the engineering design is completed, a prototype of the system is made and examined several times to verify whether it satisfies the design requirements. But in spite of this systematic procedure of design and manufacturing, defective products that do not satisfy the design requirements are still likely to exist. The reason for this is the existence of uncertainty in the design and manufacturing procedure. Such uncertainty is caused by the physical characteristics of material, exactness of measuring, manufacturing equipment, temperature or humidity at the workplace, among others. In industrial fields, there have been many recent attempts to maximize profits while enhancing the quality of the products. The key to this is considering uncertainty in the early design stage. Reliability analysis, one such attempt, is a statistical method that considers the uncertainty of design variables and calculates the probability of failure of the mechanical and structural system. The probability of failure means the probability that the system response does not satisfy given design requirements. There are three major methods for reliability analysis – the fast probability integration method, the sampling method, and the moment method. The fast probability integration method includes the first order reliability method (FORM) [1] and the second order reliability method (SORM) [2]. The sampling method includes the Monte-Carlo sampling (MCS) method and the importance sampling method [3]. The mo-

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ment me-thod calculates the probability of failure of the system by using the statistical moment of the system response function and its corresponding statistical information [4]. The moment method is divided into the full factorial moment method (FFMM) [5] and the response surface moment method (RSMM) [6]. While FFMM uses full factorial design (FFD) of design of experiments (DOE) for sampling the experimental points, RSMM uses the second order polynomial response surface model (RSM) to select the experimental points. So, the process of constructing and updating the RSM is the most important part of RSMM. RSMM updates RSM by adding a cross product term to the second order polynomial, but it has limitations in the update process of RSM. To predict the probability distribution of the system response, both FFMM and RSMM use the Pearson distribution system [7, 8].

In this research, we propose the enhanced RSMM (RSMM+), modifying the update process of RSM in RSMM. In this paper, the parts of RSMM are described first, and then we explain the modified parts of RSMM+. Finally, we apply the proposed method to test problems and survey and analyze the results.

## 2. RSMM

### *2.1 Deciding the levels and weights for each design variable*

In RSMM, three levels ($l_{i\cdot1}, l_{i\cdot2}, l_{i\cdot3}$) and three weight values ($w_{i\cdot1}, w_{i\cdot2}, w_{i\cdot3}$) for each design variable are decided. To obtain the values of the levels and weights, we compose, below, nonlinear simultaneous equations derived from the $k^{\text{th}}$ order central moment equation. The value of $k$ is from 0 to 5, and $n$ means the number of design variables.

$$w_{i\cdot1} + w_{i\cdot2} + w_{i\cdot3} = 1 \qquad (i = 1, 2, \cdots, n) \tag{1}$$

$$w_{i\cdot1}l_{i\cdot1} + w_{i\cdot2}l_{i\cdot2} + w_{i\cdot3}l_{i\cdot3} = \mu_{x_i} \tag{2}$$

$$w_{i\cdot1}(l_{i\cdot1} - \mu_{x_i})^2 + w_{i\cdot2}(l_{i\cdot2} - \mu_{x_i})^2 + w_{i\cdot3}(l_{i\cdot3} - \mu_{x_i})^2 = \sigma_{x_i}^2 \tag{3}$$

$$w_{i\cdot1}(l_{i\cdot1} - \mu_{x_i})^3 + w_{i\cdot2}(l_{i\cdot2} - \mu_{x_i})^3 + w_{i\cdot3}(l_{i\cdot3} - \mu_{x_i})^3 = \sigma_{x_i}^3\sqrt{\beta_{1x_i}} \tag{4}$$

$$w_{i\cdot1}(l_{i\cdot1} - \mu_{x_i})^4 + w_{i\cdot2}(l_{i\cdot2} - \mu_{x_i})^4 + w_{i\cdot3}(l_{i\cdot3} - \mu_{x_i})^4 = \sigma_{x_i}^4\beta_{2x_i} \tag{5}$$

$$w_{i\cdot1}(l_{i\cdot1} - \mu_{x_i})^5 + w_{i\cdot2}(l_{i\cdot2} - \mu_{x_i})^5 + w_{i\cdot3}(l_{i\cdot3} - \mu_{x_i})^5 = M_{5x_i} \tag{6}$$

The solutions of nonlinear simultaneous equations are the values of levels and weights, and they can be calculated by the numerical method. The values of levels and weights for the other variables are calculated the same way. Finally, $3^n$ experimental points are set by compounding three levels of each variable.

### *2.2 Construction of the initial RSM*

After the experimental points are determined, experiments are conducted to calculate system responses at the experimental points. First, the system response for $2n$+1 experimental points that lay on the 2nd-level of each variable are calculated, and the initial RSM for the system response function is built from this value by using the least squares method (LSM) as shown in Eq. (7).

$$\begin{aligned}\tilde{g}(\mathbf{x}) &= a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2 \\ &= \bar{a} + \sum_{i=1}^{n} \bar{b}_i \xi_i + \sum_{i=1}^{n} \bar{c}_i \xi_i^2 \quad \left(\xi_i = \frac{x_i - (l_{i\cdot3} + l_{i\cdot1})/2}{(l_{i\cdot3} - l_{i\cdot1})/2}\right)\end{aligned} \tag{7}$$

where $x_i$ is the $i^{\text{th}}$ design variable.

### *2.3 Calculation of statistical moments*

System responses for uncalculated experimental points are predicted by using the initial RSM. After the system responses for all experimental points are obtained, statistical moments of the system response function are calculated by the equations below:

$$\mu_g = \sum_{i_1=1}^{3} w_{1\cdot i_1} \cdots \sum_{i_n=1}^{3} w_{n\cdot i_n} g(l_{1\cdot i_1}, \cdots, l_{n\cdot i_n}) \tag{8}$$

$$\sigma_g = \left[\sum_{i_1=1}^{3} w_{1\cdot i_1} \cdots \sum_{i_n=1}^{3} w_{n\cdot i_n} \left(g(l_{1\cdot i_1}, \cdots, l_{n\cdot i_n}) - \mu_g\right)^2\right]^{1/2} \tag{9}$$

$$\sqrt{\beta_1}_g = \left[\sum_{i_1=1}^{3} w_{1\cdot i_1} \cdots \sum_{i_n=1}^{3} w_{n\cdot i_n} \left(g(l_{1\cdot i_1}, \cdots, l_{n\cdot i_n}) - \mu_g\right)^3\right] \Big/ \sigma_g^3 \tag{10}$$

$$\beta_{2g} = \left[\sum_{i_1=1}^{3} w_{1\cdot i_1} \cdots \sum_{i_n=1}^{3} w_{n\cdot i_n} \left(g(l_{1\cdot i_1}, \cdots, l_{n\cdot i_n}) - \mu_g\right)^4\right] \Big/ \sigma_g^4 \tag{11}$$

where each of $\mu_g$, $\sigma_g$, $\sqrt{\beta_1}_g$, and $\beta_{2g}$ represents the mean, standard deviation, skewness coefficient, and kurtosis coefficient, respectively.

### *2.4 Prediction of probability of failure*

Coefficients of the differential equation used in the Pearson distribution system are calculated from statistical moments for the system response function. The values of $a$, $c_0$, $c_1$, and $c_2$ in the equation below are coefficients of the Pearson distribution system.

$$\frac{1}{f(\tilde{g})} \cdot \frac{df(\tilde{g})}{d\tilde{g}} = -\frac{a + \tilde{g}}{c_0 + c_1\tilde{g} + c_2\tilde{g}^2} \tag{12}$$

where $\tilde{g}$ is the RSM of the corresponding system response function.

The probability distribution of the system response is predicted from the previous differential equation. Finally, the probability of failure is obtained from the Pearson distribution system.

### 2.5 Updating process of the RSM

#### 2.5.1 Calculation of the influence index value

In RSMM, the influence index value is used to select additional points. The influence index of the $i^{th}$ candidate point is calculated from the equation below.

$$\begin{aligned}\kappa_i &= \left|\frac{dP_f}{d\tilde{g}(\mathbf{x}_i)}\right| \\ &\simeq \left|\frac{\Delta P_f}{\Delta \mu_g}\cdot\frac{d\mu_g}{d\tilde{g}(\mathbf{x}_i)} + \frac{\Delta P_f}{\Delta \sigma_g}\cdot\frac{d\sigma_g}{d\tilde{g}(\mathbf{x}_i)}\right. \\ &\quad \left. + \frac{\Delta P_f}{\Delta\sqrt{\beta_{1g}}}\cdot\frac{d\sqrt{\beta_{1g}}}{d\tilde{g}(\mathbf{x}_i)} + \frac{\Delta P_f}{\Delta \beta_{2g}}\cdot\frac{d\beta_{2g}}{d\tilde{g}(\mathbf{x}_i)}\right|\end{aligned} \tag{13}$$

where $P_f$ means the probability of failure.

The influence index is the size of the deviation in probability of failure due to an additional experimental point. So the experimental point that has the maximum influence index is selected and an experiment is conducted for it.

#### 2.5.2 Listing the candidate cross product terms

When one experimental point is added, one cross product term is added. We distinguish the levels of each variable that constitute the additional experimental point. If there are $2^{nd}$-level design variables, they are excluded from the pool of the variables to make additional cross product terms. The reason for this exclusion is that $2^{nd}$-level design variables in the cross product term can cause the singularity and ill-conditioning problem in the process of the LSM. After the exclusion process, we list all potential cross product terms by compounding the remaining variables.

#### 2.5.3 Calculation of the sum of squares error

We construct a temporary RSM containing each cross product term candidate. For each temporary RSM, we predict the system responses at experimental points already calculated and estimate the sum of squares error (SSE) by comparing calculated values and predicted values. We find the cross product term that has the smallest SSE, and update the RSM by adding it.

$$\begin{aligned}\tilde{g}(\mathbf{x}) &= a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2 + \sum_{k=1}^{nmix} d_k x_{i(k)} x_{j(k)} \\ &= \bar{a} + \sum_{i=1}^{n} \bar{b}_i \xi_i + \sum_{i=1}^{n} \bar{c}_i \xi_i^2 + \sum_{k=1}^{nmix} \bar{d}_k \xi_{i(k)} \xi_{j(k)}\end{aligned} \tag{14}$$

In the above equation, nmix is the number of additional cross product terms.

#### 2.5.4 Calculation of the coefficient sum

If the SSE values of some temporary RSMs are equal to one another, we calculate the coefficient sum (CS) for each case. The CS measurement evaluates how effective the design variable is for the RSM. For the RSM we find the cross product term that has the largest CS, and we update the RSM by adding the one found. The equation to calculate the CS is shown below.

$$CS_{ij} = \left|\bar{b}_i\right| + \left|\bar{c}_i\right| + \left|\bar{b}_j\right| + \left|\bar{c}_j\right| \tag{15}$$

### 2.6 Probability of failure

The probability of failure is calculated by using the updated RSM. The previous updating process of the RSM is repeated until the probability of failure converges. If the relative errors of the probability of failure between consecutive three iterations are less than the predetermined small value, it is assumed to be converged.

## 3. Enhanced RSMM

In the proposed method, we modify the updating process of the RSM.

### 3.1 Updating process of the RSM

#### 3.1.1 Calculation of the influence index value

The influence index of RSMM+ is calculated the same way as for RSMM. We select the most influential point, but we delay the experimentation for it.

#### 3.1.2 Listing the candidate cross product terms

We distinguish the levels of each variable that constitute the additional experimental point the same way as in the RSMM. In the first updating process, if there are $2^{nd}$-level design variables, they are excluded from the pool of variables to make additional cross product terms. This is the same as the RSMM, too. But from the second updating process on, the $2^{nd}$-level design variables are not excluded from the pool of variables. The reason why the $2^{nd}$-level design variables are

$$
\begin{array}{r|ccccccccccc|l}
 & 1 & \hat{\xi}_1 & \hat{\xi}_2 & \hat{\xi}_3 & \hat{\xi}_4 & \hat{\xi}_1^2 & \hat{\xi}_2^2 & \hat{\xi}_3^2 & \hat{\xi}_4^2 & \hat{\xi}_1\hat{\xi}_3 & \hat{\xi}_3\hat{\xi}_4 & \\
\mathbf{x}_1: & 1 & -1 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & \\
\mathbf{x}_2: & 1 & 0 & -1 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & \\
\mathbf{x}_3: & 1 & 0 & 0 & -1 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & \\
\mathbf{x}_4: & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & +1 & 0 & 0 & \\
\mathbf{x}_5: & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
\mathbf{x}_6: & 1 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & +1 & 0 & 0 & = \mathbf{X}_d \\
\mathbf{x}_7: & 1 & 0 & 0 & +1 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & \\
\mathbf{x}_8: & 1 & 0 & +1 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & \\
\mathbf{x}_9: & 1 & +1 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & \\
\mathbf{x}_{10}: & 1 & +1 & 0 & +1 & +1 & +1 & 0 & +1 & +1 & +1 & +1 & \\
\mathbf{x}_{11}: & 1 & +1 & +1 & +1 & 0 & +1 & +1 & +1 & 0 & +1 & 0 &
\end{array}
$$

Fig. 1. Design matrix.

included in the RSMM+ is that they do not always cause the singularity and ill-conditioning problem. This is explained in Fig. 1 for the case of four design variables.

The term $x_k$ is the $k^{\text{th}}$ experimental point and $\mathbf{X}_d$ is the design matrix used in the LSM. The elements of the $k^{\text{th}}$ row of the design matrix are determined from the corresponding $k^{\text{th}}$ experimental point. In the design matrix, a $1^{\text{st}}$-level variable is transformed to -1, and $2^{\text{nd}}$- and $3^{\text{rd}}$-level variables are transformed to 0 and 1, respectively. Fig. 1 shows the design matrix when there are $2n$+1 initial experiments and an additional two experiments are executed. The cross product term $x_1x_3$ is added in the first updating process, and $x_3x_4$ is added in the second updating process. As indicated in Fig. 1, although the $2^{\text{nd}}$-level design variable $x_4$ constitutes the cross product term in the second updating process, it does not cause the singularity and ill-conditioning problem.

### *3.1.3 Calculation of the fisher information matrix*

In the RSMM, constructing RSM using the LSM means calculating the coefficients of a second order polynomial using the LSM. Because the LSM uses the inverse matrix of the Fisher information matrix, it cannot obtain the coefficients or obtain inaccurate coefficients when the Fisher information matrix contains the singularity and ill-conditioning problem. This means it cannot construct the RSM or construct an inaccurate RSM.

So we reselect the potential cross product terms so that the Fisher information matrix does not cause the singularity and ill-conditioning problem. We use the value of the determinant and the condition number of the Fisher information matrix to determine whether it causes such a problem or not.

If there are no candidates for the cross product term, we reselect the next influential point and repeat the previous updating process. We can prevent unnecessary experimentation with this approach.

### *3.1.4 Calculation of relative error for the probability of failure*

We construct temporary RSMs containing each reselected candidate for the cross product term. For each temporary RSM, we calculate the value of probability of failure, $P_{f,Cross,}$ by the same procedure as RSMM and then evaluate the relative error for $P_{f,Cross}$ by comparing it with the former value. Comparing each relative error, we find the largest relative error and the corresponding cross product term and update the RSM by adding this cross product term. The use of relative error for finding additional cross product terms and the use of the influence index for finding additional experimental points are analogous to each other.

$$
Error_{R,Cross}{}^{i} = \frac{\left| P_f^{(i-1)} - P_{f,Cross}^{(i)} \right|}{P_f^{(i-1)}} \times 100[\%] \tag{16}
$$

In the equation above, $Error_{R,Cross}{}^{i}$ means the relative error at the $i^{\text{th}}$ updating process.

### *3.2 Probability of failure*

The previous updating process of the RSM is repeated until the probability of failure converges.

## 4. Test problems

The results of reliability analysis using RSMM+ are compared with those of RSMM and FFMM in terms of efficiency and accuracy. The results of the MCS are presented to examine the accuracy of such methods. The test problems are shown below.

### *4.1 Fortini's clutch*

This problem is a mechanical problem having four design variables. Fig. 2 represents the physical features of the variables, and the statistical information of each variable is in Table 1. The system response function is shown in Eq. (17).

$$
y(\mathbf{x}) = cos^{-1}\left( \frac{x_1 + 0.5(x_2 + x_3)}{x_4 - 0.5(x_2 + x_3)} \right) \tag{17}
$$

### 4.2 Truss structure

This problem is a truss structure with 23 members and it has 10 design variables. Fig. 3 shows the physical features of the variables and Table 2 shows their statistical information.

$$g(\mathbf{x}) = 11 - |DISP1| \tag{18}$$

In Eq. (10), DISP1 is the deflection at PNT 1.

Table 1. Design variables in example 1.

| Variable | Distribution | Mean | S.D. | Parameters for $x_1, x_4$ |
|---|---|---|---|---|
| $x_1$ | Beta | 55.29 | 0.0793 | $\gamma_1 = \eta_1 = 5.0$ |
| $x_2$ | Normal | 22.86 | 0.0043 | $55.0269 \le x_1 \le 55.5531$ |
| $x_3$ | Normal | 22.86 | 0.0043 | $\hat{\sigma}_4 = 0.1211$ |
| $x_4$ | Rayleigh | 101.60 | 0.0793 | $x_4 \ge 101.45$ |

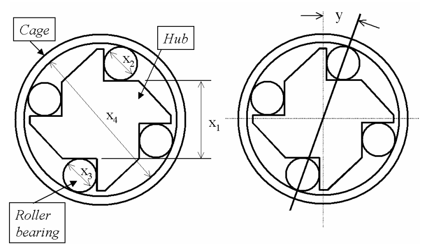


Fig. 2. Fortini's clutch.

Table 2. Design variables in example 2.

| Variable | | Distribution | Mean | S.D. |
|---|---|---|---|---|
| 1 | $E_1$ | Log-Normal | 2,1000,000 | 210,000 |
| 2 | $E_2$ | Log-Normal | 2,1000,000 | 210,000 |
| 3 | $A_1$ | Log-Normal | 20 | 2 |
| 4 | $A_2$ | Log-Normal | 10 | 1 |
| 5 | $P_1$ | Gumbel | 5,000 | 750 |
| 6 | $P_2$ | Gumbel | 5,000 | 750 |
| 7 | $P_3$ | Gumbel | 5,000 | 750 |
| 8 | $P_4$ | Gumbel | 5,000 | 750 |
| 9 | $P_5$ | Gumbel | 5,000 | 750 |
| 10 | $P_6$ | Gumbel | 5,000 | 750 |

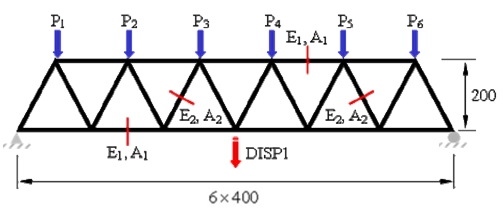


Fig. 3. Truss structure with 23 members.

### 4.3 The results of test problems

The results of the mechanical problem are presented in Table 3, and the results of the structural problem are in Table 4.

The FFMM is a basis of both the proposed method and the RSMM, so the proposed method and the RSMM cannot be more accurate than the FFMM. Therefore, we evaluate the accuracy of the proposed method and the RSMM by comparing them with the results of the FFMM. Since the largest difference of accuracy between the proposed method and the RSMM is about 1.9%, we consider the difference of accuracy between the two methods to be insignificant.

The efficiency results of the proposed method and the RSMM are shown in Fig. 4 and Fig. 5. The values above the bar graphs represent a decrease ratio on

Table 3. Results of example 1.

| | RSMM | RSMM+ | FFMM | MCS |
|---|---|---|---|---|
| $\mu_y$ | 0.12194 | 0.12194 | 0.12193 | 0.12193 |
| $\sigma_y$ | 0.01162 | 0.01162 | 0.01169 | 0.01169 |
| $\sqrt{\beta_{1y}}$ | -0.10761 | -0.10711 | -0.05766 | -0.05159 |
| $\beta_{2y}$ | 2.84167 | 2.84130 | 2.92150 | 2.88100 |
| $\Pr[y<5°]$<br>Error(%)* | 0.00149(22)<br>5.6962 | 0.00146(15)<br>7.5949 | 0.00158(81) | 0.00129 |
| $\Pr[y<6°]$<br>Error(%) | 0.07335(13)<br>1.0888 | 0.07333(11)<br>1.0612 | 0.07256(81) | 0.07392 |
| $\Pr[y<7°]$<br>Error(%) | 0.50485(13)<br>0.1110 | 0.50456(11)<br>0.0535 | 0.50429(81) | 0.50316 |
| $\Pr[y<8°]$<br>Error(%) | 0.93595(12)<br>0.0320 | 0.93594(11)<br>0.0331 | 0.93625(81) | 0.93673 |
| $\Pr[y<9°]$<br>Error(%) | 0.99929(12)<br>0.0040 | 0.99928(11)<br>0.0030 | 0.99925(81) | 0.99919 |
| $\Pr[5°<y<9°]$<br>Error(%) | 0.99780<br>0.0130 | 0.99782<br>0.0150 | 0.99767 | 0.99790 |

( ) : The Number of Function Calls
*: compared with the FFMM
(MCS : 1,000,000 Sampling)

Table 4. Results of example 2.

| | RSMM | RSMM+ | FFMM | MCS |
|---|---|---|---|---|
| $\mu_g$ | 3.06528 | 3.06060 | 3.05984 | 3.06624 |
| $\sigma_g$ | 1.09725 | 1.09880 | 1.10916 | 1.10709 |
| $\sqrt{\beta_{1g}}$ | -0.57080 | -0.56879 | -0.49888 | -0.47886 |
| $\beta_{2g}$ | 3.41199 | 3.40930 | 3.42889 | 3.38262 |
| $\Pr[g<0]$<br>Error(%)* | 0.00880(45)<br>0.2268 | 0.00891(35)<br>1.0204 | 0.00882 | 0.00833 |

( ) : The Number of Function Calls
*: compared with the FFMM
Function Calls of FFMM : 59049(310)
(MCS : 100,000 Sampling)

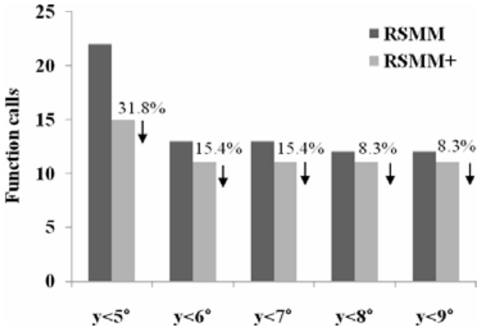


Fig. 4. Function calls in example 1.

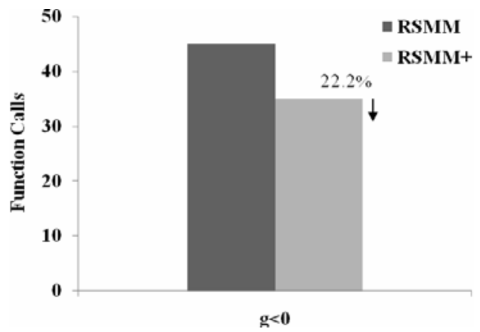


Fig. 5. Function calls in example 2.

function evaluation in the proposed method compared with the RSMM. From these results, we recognize the efficiency of the proposed method is superior to the RSMM by 8.3-31.8%.

## 5. Conclusions

In this research, we propose the RSMM+ to modify the update process of RSM in RSMM. The robustness and efficiency is enhanced in the proposed method because the feasibility of the updated RSM is estimated before an additional experiment is conducted.

We examined the results of the proposed method and RSMM for mechanical and structural test problems. Then, we demonstrated the proposed method gives 8.3-31.8% better results in efficiency, and the difference in accuracy between the proposed method and the RSMM is less than 1.9%.

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